

# An Analytical Formula of Microbending Sensitivity for Reduced Diameter Optical Fibers 

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#### Abstract

Thinner optical cables are required for data center networks to meet the increasing demand of communication traffic. For these cables thinner optical fibers are essential, and one of the concerns in reducing the optical fiber diameter is the increase in attenuation by microbending in the cabling process. It is necessary to adjust the optical fiber structure based on an analytical formula that systematically estimates the effects of structural parameters. Based on Cocchini's established method for conventional optical fibers with a coating diameter of $250 \mu \mathrm{~m}$, we have derived an analytical formula to accurately predict the microbending attenuation of optical fibers with a smaller diameter of $165 \mu \mathrm{~m}$ or less.


Keywords: optical fiber, reduced diameter optical fibers, microbending attenuation, analysis

## 1. Introduction

With the spread of cloud computing and 5 G , data centers have been actively constructed and added around the world. In the construction of data centers, it is strongly required to build economical optical networks with high space efficiency. For example, outside plant cables, which connect data centers, are installed in underground conduits. ${ }^{* 1}$ To effectively utilize the space, it is required to reduce the diameter and increase the density of optical cables. According to a report, the characteristics of higher fiber count optical cables using small-diameter optical fibers with an outer diameter of less than $165 \mu \mathrm{~m}$ meet such demand. ${ }^{(1)}$ This report pointed out that the reduction of the optical fiber diameter from $200 \mu \mathrm{~m}$ to $165 \mu \mathrm{~m}$ results in an increase in optical attenuation due to microbending, namely, microbending loss (hereinafter referred to as "MB loss"). MB loss is an important index for transmission loss characteristics of optical fiber cables.

There are two types of structures to reduce the diameter of optical fibers, as shown in Fig. 1. One structure is to reduce the coating thickness while maintaining the diameter of the cladding, which contains the core that serves as the optical waveguide, at $125 \mu \mathrm{~m}$. The other structure is to reduce both the diameter and coating thickness. For the former structure, four parameters in total, namely, the outer coating diameter and Young's modulus of the primary coating and secondary coating, respectively, must be taken into consideration when studying the structure of optical fibers. For the latter structure, five parameters in total, namely, the four parameters above and the cladding diameter, must be adjusted to determine the structure of small-diameter optical fibers while evaluating the MB loss. Determination of the optimal structure requires a technique to systematically estimate the influence of the structural parameters on the MB loss.

Cocchini derived an analytical formula for the MB loss for an optical fiber with a cladding diameter of $125 \mu \mathrm{~m}$ and an outer coating diameter of $250 \mu \mathrm{~m}$ by using the coating structure and the Young's modulus as variables. ${ }^{(2)}$ There is a report that studied the small-diameter optical
fiber structure using the formula. ${ }^{(3)}$ However, the conventional formula was derived based on the standard optical fiber structure with a cladding diameter of $125 \mu \mathrm{~m}$ and an outer coating diameter of $250 \mu \mathrm{~m}$. It does not take into consideration small-diameter optical fibers required for data centers, especially structures with an outer coating diameter of $165 \mu \mathrm{~m}$ or less, whose research has been making progress recently.

In this paper, we derive a new analytical formula for MB loss suitable for small-diameter optical fibers with an outer coating diameter of $165 \mu \mathrm{~m}$ or less by applying Cocchini's conventional technique. First, we introduce the conventional technique and the analytical formula obtained. Next, we explain the analysis conditions for small-diameter optical fibers and the analytical formula obtained. We then compare values predicted based on the analytical formula with actual measurement values, present a discussion, and provide a conclusion.


Fig. 1. Structure of small-diameter optical fibers (example)

## 2. Derivation of the Analytical Formula

## 2-1 Conventional analytical formula

MB loss $\boldsymbol{\alpha}_{\boldsymbol{M} \boldsymbol{B}}$ is expressed as Eq. (1). ${ }^{(4)}$

$$
\begin{equation*}
\boldsymbol{\alpha}_{M B} \approx A \times\left(\boldsymbol{D} / \boldsymbol{H}^{2}\right) \tag{1}
\end{equation*}
$$

Here, $A$ is the constant attributed to the optical characteristics of an optical fiber. $H$ and $D$ are the flexural rigidity and lateral rigidity of an optical fiber, respectively. $H$ can be represented by the following Eq. (2).

$$
\begin{equation*}
H \propto \pi R_{0}{ }^{4} E_{0}+\pi\left(R_{2}{ }^{4}-R_{1}{ }^{4}\right) E_{2} \tag{2}
\end{equation*}
$$

In the conventional technique, Eq. (3) was derived as the analytical formula for $D$.

$$
\begin{equation*}
D=E_{1}+\left(E_{2}-E_{1}\right)\left(\frac{E_{1}}{E_{2}}\right)^{\frac{2}{3}}\left(2 \frac{R_{2}-R_{1}}{R_{2}-R_{0}}\right)^{\frac{3}{2}} \tag{3}
\end{equation*}
$$

Here, $R_{i}$ and $E_{i}$ are the radius and the Young's modulus of the $i$ layer. As shown in Fig. 2 (a), $i=0, i=1$, and $i=2$ represent the cladding layer, primary coating layer, and secondary coating layer, respectively.

Equation (3) can be obtained based on the two steps below.
[Step 1] The optical fiber surface displacement $u_{y}{ }^{*}$ is calculated for dozens of types of optical fibers with an outer coating diameter of $250 \mu \mathrm{~m}$ whose $R_{i}$ and $E_{i}$ values are different based on the two-dimensional finite element method (hereinafter referred to as "FEM").*2 An example of the FEM calculation result is shown in Fig. 2 (b). The lateral rigidity $D$ is calculated for the result using Eq. (4) below. $F$ is the lateral pressure ( 1 MPa ), and $\theta$ is the angle to apply stress ( 0 to $9^{\circ}$ ).

$$
\begin{equation*}
D=2 F \theta R_{2} / u_{y}^{*} \tag{4}
\end{equation*}
$$



Fig. 2. (a) Structure of an optical fiber
(b) FEM calculation (example)
[Step 2] The normalized rigidity $\left(D-D_{l}\right) /\left(D_{2}-D_{l}\right)$, normalized radius $\left(R_{2}-R_{l}\right) /\left(R_{2}-R_{0}\right)$, and Young's modulus ratio $\left(E_{1} / E_{2}\right)$ are defined. Fitting is performed using the normalized rigidity as the object variable and the normal-
ized radius and Young's modulus ratio as the explanatory variables. Here, an approximation of Eq. (5) is assumed.

$$
\begin{equation*}
\frac{D-D_{1}}{D_{2}-D_{1}} \approx \frac{D-E_{1}}{E_{2}-E_{1}} \tag{5}
\end{equation*}
$$

## 2-2 New analytical formula

An analytical formula that is applied to small-diameter optical fibers with an outer coating diameter of $165 \mu \mathrm{~m}$ or less is newly derived based on the conventional technique explained in Section 2-1. For comparison, optical fibers with an outer coating diameter of $200 \mu \mathrm{~m}$, which are currently in the mainstream, are also included in the scope of application.

The FEM calculation conditions in Step 1, which are explained in Section 2-1, are shown in Table 1 based on comparison. The FEM calculation is performed on optical fiber conditions with an outer coating diameter of 110 to $210 \mu \mathrm{~m}$ (378 types in total). The optical fiber surface displacement $u_{y}{ }^{*}$ is obtained based on the results, and the lateral rigidity $D$ is calculated based on Eq. (4).

Table 1. Comparison of FEM calculation conditions

|  | New <br> calculation conditions | Conventional <br> calculation conditions |
| :---: | :---: | :---: |
| Analysis software | MSC.Nastran 2020sp1 | PDE/PROTRAN |
| Cladding diameter $(\mu \mathrm{m})$ | $75-130$ | 125 |
| Secondary coating diameter <br> $(\mu \mathrm{m})$ | $110-210$ | 250 |
| Primary Young's modulus <br> $(\mathrm{MPa})$ | $0.05-1.0$ | $10-20$ |
| Secondary Young's modulus <br> $(\mathrm{MPa})$ | $1000-3000$ | $500-1000$ |
| Number of calculation <br> conditions | 378 | 30 |

In Step 2, the method of setting an object variable and explanatory variables and obtaining an analytical formula based on fitting is not changed. Instead, the variables and the formula are changed. The object variable of the new analytical formula is the normalized rigidity $\left(D-D_{l}\right) /\left(D_{2}-D_{l}\right)$. The explanatory variables are $\left(R_{2}-R_{l}\right) /\left(R_{2}-R_{0}\right)$, a term dependent on the primary radius, and $\left(E_{1} / E_{2}\right)$, the Young's modulus ratio of the primary and secondary coatings, as in the case of the conventional technique, plus $\left(R_{0} / R_{2}\right)$, the radius ratio of the cladding diameter and the secondary coating diameter. An expansion formula as shown below is used.

$$
\begin{equation*}
\log _{10} \frac{D-D_{1}}{D_{2}-D_{1}} \approx \sum_{i, j, k} c_{i j k}\left(\log _{10} \frac{R_{2}-R_{1}}{R_{2}-R_{0}}\right)^{i}\left(\log _{10} \frac{E_{1}}{E_{2}}\right)^{j}\left(\log _{10} \frac{R_{0}}{R_{2}}\right)^{k} \ldots \tag{6}
\end{equation*}
$$

Here, $i, j$, and $k$ are integers from 0 to 2 , and $c_{i j k}$ is a constant. Approximation of Eq. (5) based on the calculation conditions was invalid. Thus, Eq. (7) is newly used.

$$
\begin{equation*}
\frac{D_{1}}{E_{1}}=\frac{D_{2}}{E_{2}} \approx \frac{c_{1}}{\left(1-\frac{R_{0}}{R_{2}}\right)^{c_{2}}}+c_{3} \tag{7}
\end{equation*}
$$

Table 2. $c_{i j k}$ and $c_{1}$ to $c_{3}$ values

| $c_{000}$ | -0.612 |  | $c_{1}$ | 0.209 |
| :---: | :---: | :---: | :---: | :---: |
| $c_{100}$ | 3.62 |  | $c_{2}$ | 1.21 |
| $c_{010}$ | 0.253 |  | 0.401 |  |
| $c_{001}$ | -7.13 |  |  |  |
| $c_{200}$ | 0.788 |  |  |  |
| $c_{110}$ | 0.329 |  |  |  |
| $c_{101}$ | 2.32 |  |  |  |
| $c_{020}$ | -0.0620 |  |  |  |
| $c_{011}$ | -0.986 |  |  |  |
| $c_{002}$ | -8.70 |  |  |  |

Table 2 shows the values of constant $c_{i j k}$ and $c_{l}$ to $c_{3}$ obtained by fitting.

By using Eq. (6) and Table 2, $D$ of an optical fiber with an outer coating diameter of $165 \mu \mathrm{~m}$ or less can be calculated using arbitrary parameter values, including the cladding diameter as a new value. $D / H^{2}$, the second term on the right side of Eq. (1), which can be calculated by $H$ in Eq. (2) and $D$ in Eq. (6), is calculated, and the standard value is defined as the microbending sensitivity (hereinafter referred to as "MBS"). The MBS is normalized using $D / H^{2}$ of an optical fiber with a cladding diameter of 125 $\mu \mathrm{m}$ and an outer coating diameter of $200 \mu \mathrm{~m}$ as the standard. Similarly, regarding the MB loss in the actual measurement, the relative value using the optical fiber structure with an outer coating diameter of $200 \mu \mathrm{~m}$ as the standard is set as the MBS.

## 3. Experiment

## 3-1 Optical fibers fabrication

The preform for optical fibers was melted in a heating furnace and drawn into cladding fibers with a diameter of 80 to $125 \mu \mathrm{~m}$. After applying two layers of an ultraviolet curable resin and allowing them to cure, an optical fiber was wound on a bobbin to obtain an optical fiber sample.

Details of the optical fiber samples fabricated are shown in Table 3. The samples are changed on three levels to investigate the dependence of MBS on the cladding diameter or the Young's modulus of the primary coating.

Table 3. Details of the optical fiber samples

| Cladding diameter ( $\mu \mathrm{m}$ ) | 80 |  |  | 100 |  |  | 125 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Primary coating diameter ( $\mu \mathrm{m}$ ) | 135 |  |  |  |  |  |  |  |  |
| Secondary coating diameter ( $\mu \mathrm{m}$ ) | 165 |  |  |  |  |  |  |  |  |
| Primary Young's modulus (MPa) | 0.2 | 0.4 | 0.9 | 0.2 | 0.4 | 0.9 | 0.2 | 0.4 | 0.9 |
| Secondary Young's modulus (MPa) | 1500 |  |  |  |  |  |  |  |  |

## 3-2 Measurement of the MB loss

Figure 3 shows a schematic diagram of the MB loss measurement. The measurement is conducted in accordance with Method B of IEC TR 62221. A piece of sand-
paper with a surface roughness of \#240 is pasted on a bobbin with an inner diameter of 280 mm . An optical fiber sample 600 m long is wound with a tension of 80 g , and the transmission loss is measured at a wavelength of 1,550 nm . The difference in the transmission loss before and after winding is regarded as the MB loss.


Fig. 3. Schematic diagram of the MB loss measurement

## 4. Results of Analysis

## 4-1 Cladding diameter dependence

The dependence of MBS on the cladding diameter $\left(2 R_{0}\right)$, calculated based on Eq. (3), which is used for conventional analysis, and Eq. (6), which is used for new analysis, is shown in Fig. 4. Both formulas show that the higher the $2 R_{0}$, the lower the MBS. In regions where $2 R_{0}$ is small, the new analytical formula indicates an MBS lower than that of the conventional formula. The MBS of these formulas reverses at around $2 R_{0}=120 \mu \mathrm{~m}$. The new analytical formula indicates a minimum value of $2 R_{0}=125 \mu \mathrm{~m}$.


Fig. 4. Cladding diameter dependence of MBS

## 4-2 Primary coating diameter dependence

Figure 5 shows the dependence of MBS on the primary coating diameter $\left(2 R_{l}\right)$ stratified by the cladding diameter $\left(2 R_{0}\right)$. With $2 R_{0}=125 \mu \mathrm{~m}$, the MBS tends to decrease with the increase in $2 R_{l}$. There is no significant difference between the conventional formula and the new analytical formula. Meanwhile, with $2 R_{0}=80 \mu \mathrm{~m}$, the MBS tends to monotonically decrease with the increase in $2 R_{l}$ based on the conventional formula and the new analytical formula. However, the MBS increases with $2 R_{l}=150$ $\mu \mathrm{m}$ based on the new analytical formula.


Fig. 5. Primary coating diameter dependence of MBS

## 4-3 Primary Young's modulus dependence

The dependence of MBS on the Young's modulus of the primary coating $\left(E_{I}\right)$ stratified by the cladding diameter ( $2 R_{0}$ ) is shown in Fig. 6. With $2 R_{0}=80 \mu \mathrm{~m}$ and $125 \mu \mathrm{~m}$, the MBS tends to decrease with the decrease in $E_{1}$.

With $2 R_{0}=80 \mu \mathrm{~m}$, the new analytical formula indicates an MBS lower than that of the conventional formula in regions where $E_{l}$ is small. With $2 R_{0}=125 \mu \mathrm{~m}$, the MBS decreases to about $1 / 3$ with the decrease in $E_{I}$ from 1.0 MPa to 0.2 MPa . Meanwhile, with $2 R_{0}=80 \mu \mathrm{~m}$, the MBS decreases to about $1 / 4$, indicating that the reduction effect is greater than that of $2 R_{0}=125 \mu \mathrm{~m}$. The MBS is significantly influenced by $E_{l}$ when $2 R_{0}$ is small.


Fig. 6. Primary Young's modulus dependence of MBS

## 5. Discussion

## 5-1 Comparison of Young's modulus dependence between calculated and experimental results

To confirm the validity of the new analytical formula, we compare the calculation values of the MBS with actual measurement values.

The dependence of MBS on the Young's modulus of the primary coating $\left(E_{l}\right)$ is stratified based on the cladding diameter $\left(2 R_{0}\right)$ in Fig. 7. With $2 R_{0}=80 \mu \mathrm{~m}$, the calculation and the actual measurement show a similar trend. With $2 R_{0}$ $=125 \mu \mathrm{~m}$, the calculation values of the MBS almost triple as $E_{l}$ increases from 0.2 MPa to 0.9 MPa , whereas actual measurement values increased by only about 1.1 times.

Thus, there is a difference between the calculation and the actual measurement. This is attributable to the fact that, because the primary coating is very thin $(5 \mu \mathrm{~m})$ with $2 R_{0}=$ $125 \mu \mathrm{~m}$, the buffer effect against microbending is smaller than expected based on calculation due to the decrease in $E_{1}$.


Fig. 7. Comparison of measured and experimental values in primary Young's modulus dependence

## 5-2 Comparison of cladding diameter dependence between calculated and experimental results

Figure 8 shows the dependence of calculation values and actual measurement values of the MBS on the cladding diameter $\left(2 R_{0}\right)$ based on the Young's modulus of the primary coating $\left(E_{l}\right)$.

With $E_{I}=0.9 \mathrm{MPa}$, the trend of actual measurement matches that of calculation. However, the results with $E_{1}=$ 0.2 MPa are significantly different. In calculation, the MBS shows a trend of monotonic decrease with the increase in $2 R_{0}$. Actual measurement shows a trend of upward convex with a peak of $2 R_{0}=100 \mu \mathrm{~m}$. The cause behind these phenomena is currently unknown, and further investigation will be required.


Fig. 8. Comparison of measured and experimental values in cladding diameter dependence

## 6. Conclusion

We applied Cocchini's technique to derive a new analytical formula for MB loss suitable for small-diameter optical fibers with an outer coating diameter of $165 \mu \mathrm{~m}$ or less. Compared to the conventional formula, the new analytical formula showed lower MBS in regions where the cladding diameter was small.

Based on the comparison between the calculation values and the actual measurement values using the new analytical formula, the trend of calculation values matched that of actual measurement values under conditions when the cladding diameter was $80 \mu \mathrm{~m}$ and the Young's modulus of the primary coating was 0.9 MPa . Accordingly, the new analytical formula is considered to be useful to estimate the MB loss. It should be noted that calculation values are significantly different from actual measurement values in regions where the cladding diameter is $125 \mu \mathrm{~m}$ and the Young's modulus of the primary coating is 0.2 MPa . Thus, precautions should be taken when applying the new analytical formula.

## Technical Terms

*1 Underground conduit: Used for installation of underground optical cables. Effective utilization of existing conduits is considered important because installation and expansion of new conduits incur huge costs.
*2 Finite element method (FEM): An analysis technique to conduct numerical analysis by splitting a structure into a finite number of elements (mesh). When a load is applied to a calculation target, many elements that form an object are individually transformed, making it possible to reproduce transformation of the object.

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